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FURTHER EVIDENCE OF THE TIME SERIES PROPERTIES OF ACCOUNTING INCOME

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I. INTRODUCTION

THE BEHAVIOR OF ACCOUNTING INCOME TIME-SERIES is of considerable interest to researchers in accounting, finance, and related disciplines. For example, the properties of accounting income time-series are directly related to accounting questions of management manipulation of accounting income and interim reporting. However, increased knowledge of income time-series behavior will be of most benefit to the extent that it contributes to the improvement of models in finance and improvement of the quality of accounting income numbers that are variables in predictive models [4], [5], [6]. Beaver compiled an impressive list of studies that required assumptions concerning the time-series behavior of accounting income or used accounting income as a predictive variable. The studies included in the Beaver list related to: valuation models of the firm, valuation of firm securities, dividend policies, earnings growth rate forecasts, evaluation of the informational content of accounting numbers, forecasting the failure of firms, and industrial concentration and accounting rates of return [6, p. 64].

Most previous research provided convincing evidence that income changes are independent [1], [2], [8], [18], [19]. Without exception, these studies have examined the total sample of time series selected for study. However, earlier work by Brooks and Buckmaster [9], [12] and Beaver [6] suggests that some subsets of series that are homogeneous with respect to configuration behave in a different manner than other income time-series. Specifically, the results of these studies suggested that series containing what appear to be non-systematic disturbances or shocks have special characteristics. These characteristics are obscured by the averaging process when the sample is not stratified.

Strong evidence has been provided by Ball and Watts [2] that income time-series follow a submartingale or some similar process. This present study identifies systematic conditions where income time-series do not follow a submartingale process. This has substantial impact on conclusions stated in previous studies concerning firm survival, effectiveness of income manipulation, and efficient market pricing.

The tests and results presented in this study are exploratory and descriptive. Thus, they are intended only as an incremental step towards the development of a general theory of income time-series behavior.

The approach used in this study was to stratify the sample according to distance of a given observation from an operationally defined normal income for each company. Iterative procedures were then used to determine the best exponential smoothing model and smoothing constant (α) for each stratum. This procedure was performed for each of three separate stratification rules. The outcome of our tests

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suggests that the series included in the outer strata (from the norm) do not follow a martingale process. Also, the results indicate that the observations subsequent to the outer strata classificatory observations tend to revert to the income levels that preceded the classificatory observation.

II. Prior Evidence

The authors are aware of four studies that have been primarily concerned with examining the properties of accounting income time-series. One of these, Brown and Ball [10], was unrelated to the problem being investigated in this paper. The object of the Ball and Brown research was to identify the portions of firm income that can be associated with the economy and the industry to which the firm belongs.

Beaver [6] conducted an exploratory study on accounting rates of return series. His primary conclusion of relevance to this research was that these measures tended to be mean reverting. He was careful, however, to point out that undeflated income series do not necessarily behave in the same manner as rates of return series.

Ball and Watts [2] concentrated on examining the dependence in undeflated income time-series. Earlier research that had used income time-series as predictor model inputs seemingly established that there was little or no dependence in the series.[1] Much of the evidence obtained from these studies was derived from examination of changes in income. Ball and Watts, since their interest in the behavior of income series was primary, added exponential smoothing models as their primary analytical tool. Their study indicated that, in general, accounting income follows a submartingale process. That is, the best predictor of period "t" income is the income of period $t-1$.

Buckmaster and Brooks [12] also were primarily concerned with the time-series behavior of accounting income and utilized exponential models as the primary analytical tool. This study was, however, very limited in both scope and objective. The objective was to determine if the income of companies taking what appeared to be a "financial bath" returned to previous income levels. The sample consisted of compustat firms that had: (1) observations of both operating income and extraordinary item series that were extremely low relative to the *ex-post* linear structure, and (2) the extremely low observations occurred in the same accounting period. The investigators found that: (1) in the year subsequent to the extremely low observations, both operating income and extraordinary items tended to move back up towards the levels preceding the period containing the extremely low observations, and (2) the best predictions of operating income in the period following the low observations were provided by the first order smoothing model with a smoothing constant (α) of .333.

III. Some Implications of Martingale or Similar Processes

Ball and Watts identify two implications of their study which arise from the evidence that time-series follow a martingale or similar process: (1) Conceptually,

1. For a review of these studies, see [2], pp. 666–667.

every firm can be expected to fail; and (2) "Income Smoothing" efforts by management cannot be successful. They describe why firms can be expected to fail by stating:

> For a martingale (that is, ignoring trend), there is a finite probability, which is a function of the variability of the process, that the expectation of income at some future time will be negative or zero, and that the firm will on average fail. The expectation of *all* future incomes is changed with each observation. Hence, investors face greater risk than under the other type of income process [a series that is not a submartingale]. The value of the firm should change in the order of a normal proportionality factor times the change in income, reflecting the changed expected profitability. [2, pp. 665–666]

Despite the significance of the above inference, Ball and Watts may have been even more concerned with the implications related to income smoothing. In very general terms, the income smoothing hypothesis is that management will attempt to minimize the volatility of the income series. The hypothesis has, however, been stated with widely varying degrees of rigor. Ball and Watts chose to discuss smoothing in its most general form. They state:

> Smoothing implies a return to good times, on average, after bad times, during which income decreases are artificially reduced by smoothing practices. It implies that many increases in income are also temporary, and can therefore be smoothed in order to avoid the impression of permanence...the smoothing of income by the accounting profession would seem to ignore the possibility that good times are not followed, on average, by bad times. A submartingale implies that a firm is stuck with good and bad times (deviations of realized incomes from expectations) when they occur since a submartingale, by definition, is a process in which any one observation becomes the basis for the expectation of the next...the behavior of the expectation of a submartingale over time would be such as to make nonsense of the notion of income smoothing. [1, p. 664]

If (1) income time-series of companies with major shifts in income levels do not follow a submartingale process, and (2) income in the year subsequent to an extremely large gain or loss tends to regress to previous levels, then the Ball and Watts statements do not hold. The validity of their conclusions relies on the nature of the independence in accounting income-series that they found.

If income does tend to improve after an extremely bad period, then it cannot necessarily be concluded that every firm can be expected, on average, to fail. If income time-series contain this characteristic, failure, and thus risk, is less than what might be expected. Conversely, we can expect that an extremely high income period is only temporary and that income is likely to shift downward toward income levels of periods preceding the high income year.

Even if series with observations in the outer strata of the population are dependent and tend to be mean regressive, these properties do not indicate that income smoothing as identified in some of the more rigorous statements of the smoothing hypothesis ([3], [7], [13], [16]) can be achieved. The existence of these properties would, however, negate the conclusion that income smoothing cannot be maintained.

The income smoothing question is quite complicated and there may be many factors that have prevented the accumulation of strong evidence of smoothing behavior. For example, Beaver pointed out that many accounting rules dictate that the "unexpected" component of firm income be averaged over several periods and subsequently found evidence that "a moving average procedure would produce a measure where these properties [serial correlation] would be obscured, even though the underlying process was mean reverting" [6, p. 81]. Furthermore, management may engage in income manipulation behavior other than smoothing. Brooks and Buckmaster [9] found that the "financial bath" may be widely practiced by those firms having an unusually large operating loss and that many firms may, at times, attempt short-run income maximization.[2] The results of this paper support the possibility that both types of behavior are practiced. These types of behavior may also tend to obscure the effects of income smoothing efforts. Certainly, if the martingale process fails to hold systematically for identifiable series, then more evidence is required to prove that income smoothing efforts are ineffectual.

IV. Methodology

The research is composed of three separate sets of tests, each set being based upon a different decision rule which is used to stratify the sample. In very general terms, the stratification rules are:

(1) A linear regression rule that stratifies by likelihood of an income observation falling a specific distance from time trended earnings estimates;

(2) A modified percentage change rule that stratifies by the percent change of a given year's income from the previous year's income; and

(3) A normalized first difference rule that stratifies according to the first difference in income divided by the standard deviation of the first differences for that firm.

Obviously, all three of the stratification rules are rather primitive. However, emphasis is directed toward determining the characteristics of different subsets of income time-series, not toward finding a superior stratification rule. One can expect that the partitioning rules will not be successful in separating observations into strata having different best predictor models if they do not systematically include some characteristic that affects prediction and, thereby, the choice of the best predictor model. The rules employed were selected because they provide crude, but operational, measures of change from previous performance levels and trends. The research discussed in Part II of this paper [9], [12] suggested that identification of companies with shifts in relative performance levels might provide productive stratification rules for examining the properties of income time-series. Our results indicate that the primitive partitioning rules, even with their serious limitations, were successful in classifying observations into subsets having different "best predictor" models.

The methodology applied to each of the three stratification rules may be separated into three phases. The sample of income time-series observations for the specific decision rule is drawn in phase one. In phase two, the stratification rule is applied to the sample in order to determine the members of each stratum. In the final phase, each stratum of observations is subjected to tests to determine the best smoothing model and the best smoothing constant for that particular stratum.

2. Also, see Copeland and Moore [14] for evidence of the financial bath and Copeland and Wodjak [15], and White [20] for evidence of income maximization or loss minimization.

A. *The Linear Regression Rule*

1. *Sample selection.* The income observations used in the study are from a July, 1974 edition of the COMPUSTAT annual industrial tape containing financial data for 2630 companies.[3] Net income after both taxes and all extraordinary items is used.

All firms having net income data reported for a sequential period of nine or more years over the period 1954 through 1973 are included in the sample. A minimum of nine years of sequential income data is required to satisfy the requirements of the linear regression stratification rule. Thus, with sufficient data, as many as twelve sets of nine-year income sequences may be obtained for any one company on the tapes. A total of 15,661 observations of nine-year income series are obtained with this procedure.

2. *Stratifying the sample.* The fifth through seventh years of each nine-year income series are used in deriving a time-trended linear regression equation.[4] Thus the postulated linear relation is

$$y_t = \alpha + \beta X_t + \mu_t,$$

where: for $t = 5,6,7,$ we have $X_t = 1,2,3$ and the observations of actual income numbers, y_t. Applying simple least squares, an estimate of income for the eighth year, $t = 8$, is derived

$$\hat{Y}_8 = \hat{\alpha} + \hat{\beta} X_8,$$

where: $\hat{\alpha}$ and $\hat{\beta}$ are the estimated model parameters based on the sample of three observations, $X_8 = 4$, and $\hat{Y}_8$ is the derived estimate of the eighth year income. A t-test statistic is obtained from the eighth year's actual income variation from the estimated income, $(Y_8 - \hat{Y}_8)$.[5] The eleven strata for this rule are defined in terms of t-test statistics and are illustrated in column 1 of Table 1. The statistic determined for the variation from the regression line in the eighth year determines in which of the strata an income series is classified.

3. *Tests for best predictor model.* Estimates of the ninth year income of each of the nine-year series provide the basis for selecting the best predictor model for each of the strata. First, second, and third order exponential smoothing models are applied to years one through eight of each nine-year time-series of each stratum to

3. The number of companies on the compustat tapes has increased greatly in the past few years, thus reducing much of the bias from size that concerned researchers in the late sixties and early seventies. Clearly, a survival bias also exists. However, our results on the extreme upper stratum indicate mean reversion and at least part of the mean reversion on the extreme lower strata are probably not due to the survival bias. The predominance of observations from the later years in the study also decreases the survival bias; i.e., some of the companies may fail.

4. The three year series was used in preference to longer time series since it explained a larger portion of the variation of the income observations about the least-square-line. Three through seventeen year trend lines were tested.

5. This partitioning method was chosen for its simplicity and computational efficiency, not for its statistical superiority. The existence of autocorrelation decreases the predictive ability of the linear regression model employed in this study. First, since it does not account for recent disturbances, it is biased. Second, the simple least-squares estimators are less efficient than estimators derived from a model considering the auto-correlated disturbance. It follows that biases in the t-statistic also exist. See [17, pp. 196–197].

derive a ninth period estimate of income.[6] Sixteen different smoothing constants are used with each of the smoothing models.[7]

The smoothing models provide a means by which the nature of the historical time dependence in the income series can be judged.[8] The degree of dependence on previous periods is indicated by the smoothing constant that generated the best predictor model. Thus, for example, with the first order smoothing model,

$$\hat{Y}_n = \alpha Y_{n-1} + (1-\alpha)\hat{Y}_{n-1},$$

if a smoothing constant equal to one, $\alpha = 1$, generated the best estimate of income, $\hat{Y}_n$; then the best predicator model will be a martingale (i.e., the most recent outcome in a time-series is the best predictor of the next outcome).[9] Greater dependence of the estimate on periods preceding the most recent outcome occurs as α decreases from one and approaches zero. When α equals zero, a process with a constant expectation is implied.

Iterative procedures were used to determine the best smoothing model and smoothing constant for each stratum. The first error measure used to select the best smoothing model and constant was the mean absolute error of the ninth year income for the I time-series of each strata

$$m = \frac{1}{I} \cdot \sum_{i=1}^{I} |Y_{i9} - \hat{Y}_{i9}|.$$

As in the Ball and Watts study [2, p. 675], the mean absolute error is chosen as the dominant error measure to avoid assumptions concerning the distribution of errors. The standard error of the estimate (SEE)

$$s = \left[\frac{\sum_{i=1}^{I} \left(Y_{i9} - \hat{Y}_{i9}\right)^2}{I-2} \right]^{1/2}$$

is calculated as a secondary error estimate on each strata. As the results will indicate, the two error measures are quite similar, though the SEE seems much more unstable in providing consistent results. Slight changes in strata boundaries (not reported) occasionally cause substantial shifts in the optimal smoothing

6. See Appendix A for a description of the three smoothing models.

7. Iterations were made for each of the smoothing models using the following α-levels: .05, .10, .20, .30, .333, .40, .45, .50, .55, .60, .65, .70, .80, .90, .95 and .999 where .999 will be considered approximately equal to 1.0.

8. Ball and Watts tested the validity of exponential smoothing models as they are used in this study. They concluded, "The partial adjustment class of forecasting models [exponential smoothing models] differentiates processes with expectations which are constant or deterministic functions of time from sub-martingale processes." [2, p. 697].

9. A sequence is a martingale if we have the expectation,

$$e(Y_{n+1} | Y_0 \cdots Y_n) = Y_n \qquad \text{for all } n$$

constant and smoothing model with the SEE measure. This condition does not occur with the absolute mean error measure.

The smoothing models are used because of their ability to provide information on the historical time dependence of income series, not because they are superior to other available time-series data fitting models. They provide a computationally feasible, and quite efficient, means to test the large samples we employed to determine if the submartingale is the optimal model over the various defined strata. The intent is directed toward determining if the submartingale does not hold over some definable subset of observations. It is beyond the scope of this paper, although quite likely the task of some future empiricist, to attempt to specify the optimal models, of the broader class of data fitting models for time-series analysis, that apply when the submartingale is non-optimal. This paper purposely restricts this initial investigation to the first, second, and third order smoothing models.

B. *A Modified Percentage Change Rule*

1. *Sample selection.* The same COMPUSTAT data used for the first partitioning model is employed. All firms having net income data reported for a sequential period of seven or more years over the 1954 through 1973 time period are included in the sample. A minimum of seven years of sequential income data is used to provide the smoothing models with a minimum of six observations. Thus, with sufficient data, as many as fourteen seven-year income sequences may be obtained for any one company. A total of 19,935 observations were obtained with the procedure.

2. *Stratifying the sample.* The fifth and sixth year of each seven-year income series are used to calculate the modified percentage change, p, in income,

$$p = \frac{Y_6 - Y_5}{|Y_5|}$$

An absolute value is used in the denominator to enable the specification of a measure of change for firms having negative income in the fifth year of an income time-series. This also will show that decreases in income always result in a negative-change number and that increases in income always result in a positive-change number. The cut-off p values used to form the strata are given in column 1 of Table 2.

A major weakness of the percent-change based model can occur since very large change numbers can exist if the fifth year income observation is near zero. The effect would be to introduce more randomness in defining extreme outer strata since these strata would contain many observations where large changes in performance, as perceived by analysts or measured by other stratification measures, had not occured. The reliability of the selected optimal model and optimal smoothing constant for the extreme strata is weakened by the distortion caused by incomes near zero. To partially control against this problem, an arbitrary cut-off point of 1600 percent, plus or minus, is used to exclude extreme changes from the sample. The footnotes in Table 2 indicate the effects of including these extreme 135 observations. Examination of the observations with this large change indicates that the base year income, year 5, is near zero for most of the observations.

3. *Tests for best predictor model.* Estimates of the seventh year income of each of the seven-year series provide the basis for selecting the best predictor model for each stratum. The same smoothing models and smoothing constants utilized with the first stratification rule are then applied to years one through six of each seven-year time series of each stratum to derive the seventh period estimate of income. The actual and estimated seventh year income are used in calculating both a mean absolute error and standard error of the estimate for each model and constant within each stratum. The error measures are in turn used to define the best smoothing model and constant for each stratum.

C. *The Normalized First Difference Stratification Rule*

1. *Sample selection.* The data source is the same one used for the two rules just discussed. A minimum of seven years of sequential income data is used so that the smoothing models will have a minimum of six observations. Thus, with sufficient data there can be fourteen possible income sequences for each company.

2. *Stratifying the sample.* This stratification rule, unlike the previous two, is based upon a varying number of observations. Each company series is stratified by a normalized first difference measure.

$$d_n = \frac{Y_n - Y_{n-1}}{\sigma_{d_{n-1}}}$$

where Y_n is the income reported in year n and $\sigma_{d_{n-1}}$ is the standard deviation of the first differences of yearly income. The standard deviation, $\sigma_{d_{n-1}}$, is calculated with the first differences of years one through n-1. At a minimum, the standard deviation would be based on the first differences of income for years one through five. Thus, for each subsequent observation in the time-series, the additional year's data is incorporated into the computation of the standard deviation of first differences. For example, a company having income data for all years in the twenty-year period would have a normalized first difference calculated for year nineteen,

$$d_{19} = \frac{Y_{19} - Y_{18}}{\sigma_{d_{18}}}$$

where $\delta_{d_{18}}$ is calculated using the first differences of years one through eighteen. The "roll forward" addition of observations in calculating the standard deviation of first differences is used in an attempt to both avoid the controversy of selecting, or searching for, an "optimal" historical time frame for calculating a variance measure. Additionally, the same size was much smaller, 10,619 observations, than with either of the previous stratification rules since data was required from the first year available on the tapes. Thus, a greater likelihood of survival bias exists with this stratification rule. The first column of Table 3 indicates the cut-off d values used to stratify the sample.

3. The testing using the normalized first difference rule is identical to that used with the modified percentage change rule.

V. Results

A. *The Linear Regression Stratification Rule*

The results of our tests for best predictor models and smoothing constants when the linear regression stratification rule is used are presented in Table 1. Column 1 describes the strata. The strata are ordered from the extremely high income observations to the extremely low observations. Column 2 describes the stratum as a percentage of the total sample and Column 3 indicates the number of observations in the stratum. Using the mean absolute error (MAE) as a measure, Column 4 indicates which of the smoothing models provides the best predictions when the best smoothing model is used with the best α-level. Column 5 indicates the best α-level for the best smoothing model in terms of lowest MAE. Columns 6 and 7 indicate the same results as column 5 and 6 except that the standard error of the estimate (SEE) is used as the error measure.

Strata on the extremes (over 35% of the sample for the MAE and 15.2% for the SEE) follow a process other than a submartingale. This outcome is contrary to the Ball and Watts outcome for their entire sample. Obviously, however, if this sample had not been stratified, the best predictor model would have been the first order model with an α-level of one. This outcome is what would be expected from earlier research.

TABLE 1

Linear Regression Stratification Rule
Parameters of the Best Prediction Model for Each Stratum

			Mean Absolute Error		Standard Error of the Estimate	
1	2	3	4	5	6	7
Likelihood of t-test Statistic	% of Sample	Sample Size	Order of Best Smoothing Model	Best Smoothing Constant for the Best Model	Order of Best Smoothing Model	Best Smoothing Constant for the Best Model
$.99 \leqslant X$	1.4	227	1	.8	1	.5
$.95 \leqslant X < .99$	5.7	889	1	1.0	1	1.0
$.9 \leqslant X < .95$	6.6	1038	1	1.0	1	1.0
$.8 \leqslant X < .9$	11.7	1828	1	1.0	1	1.0
$.7 \leqslant X < .8$	10.5	1647	1	1.0	1	1.0
$.3 < X < .7$	30.4	4765	1	1.0	1	1.0
$.2 < X \leqslant .3$	9.1	1431	1	.95	1	1.0
$.1 < X \leqslant .2$	10.8	1696	1	.9	1	1.0
$.05 < X \leqslant .1$	6.1	949	1	.45	3	.3
$.01 < X \leqslant .05$	6.2	962	2*	.3*	2	.3
$X \leqslant .01$	1.5	229	1	.5	1	.8
	100.0	15661				

* On this stratum and with this error measure the third order model with an $\alpha = .95$ gave nearly identical results. Additional interations with α near .3 and .95 were run with the minimum mean absolute error at $\alpha = .291$.

"Best" smoothing constants of less than one for extreme strata, using the MAE as a measure, show that the income in the period following a high income ($.99 \leqslant X$) or a low income [($.05 < X \leqslant .3$) and ($X \leqslant .01$)] period are best described by a process with a constant expectation.[10] Likewise, smoothing constants of less than one for the second order model for the stratum ($.01 < X \leqslant .05$) indicate a linear expectation with both error measures. For the stratum ($.05 < X \leqslant .1$) and the SEE measure, the best predictor is a third order model, and an exponential expectation exists. The mean error terms (not presented) of the first order model that had a smoothing constant of one were examined.[11] We found that in all the strata where the best model has an $\alpha < 1$ that there was a return toward income levels preceding the disturbance period. Thus, for example, the mean error for the stratum ($.05 < X \leqslant .1$) for the first order model with an $\alpha = 1$ is 2.155 million. Since the first order model with $\alpha = 1.0$ for this low income stratum had an average error of 2.155, we can conclude that the actual income exceeds the submartingale estimated income and partially reverts to earlier reported income levels.

B. *The Percentage Change Stratification Rule*

Table 2 presents the outcome of our results from applying the percentage change rule. The format of Table 2 is identical with that of Table 1. The outcome using the percentage change rule is, in most ways, almost identical to the outcome of the application of the linear regression stratification rule. Again, strata on the extremes follow a process other than a submartingale process and, if the sample is not stratified, the best predictor model for the entire sample is the first order model with an α-level of one (a submartingale).

There are, of course, some differences from the linear regression rule. First, there is a slight decrease in the percentage of the sample having a best α-level of less than one for the MAE measure. The proportion of the sample having this property was 30.4%, still quite substantial. The SEE measure defines a larger percentage, 29.7%, of the sample as being nonsubmartingale than the SEE measure with the regression stratification rule, 15.2%. This change in the percentage of the sample having a best smoothing constant of less than one between the regression and percentage stratification rule is mainly accounted for on the income decrease side; there was somewhat of a weakening in the income increase strata (a decrease in population proportion of .7% for MAE and .2% for SEE). It should also be noted that the best smoothing model for both the highest stratum and the lowest stratum is the second order model rather than the first order model as it was for the linear

10. This contention is supported by tests in a related situation [12]. The movement of operating income subsequent to an extreme deviation from trend on the low side was examined. It was found that the series tended to move back toward previous trends and that the "best" smoothing constant with the first order smoothing model was 1.0 for the average observation and was .333 in the period subsequent to the extreme deviation.

11. Where the mean error, m', for the sample of I income observations, Y_i, for each stratum and σ-level is

$$m' = \frac{1}{I} \cdot \sum_{i=1}^{I} (Y_i - \hat{Y}_i)$$

where $\hat{Y}_i$ is the predicted income.

TABLE 2

PERCENTAGE CHANGE STRATIFICATION RULE
PARAMETER OF THE BEST PREDICTION MODEL FOR EACH STRATUM

			Mean Absolute Error		Standard Error of the Estimate	
1	2	3	4	5	6	7
Percentage Change	% of Sample	Sample Size	Order of Best Smoothing Model	Best Smoothing Constant for the Best Model	Order of Best Smoothing Model	Best Smoothing Constant for the Best Model
900 < Change ⩽ 1600*	.3	66	2	.1	2	.1
600 < Change ⩽ 900	.4	77	1	.8	1	.8
400 < Change ⩽ 600	.5	108	1	1.0	1	.05
200 < Change ⩽ 400	2.1	424	1	1.0	1	1.0
100 < Change ⩽ 200	5.2	1031	1	1.0	1	1.0
0 ⩽ Change ⩽ 100	61.8	12310	1	1.0	1	1.0
−100 ⩽ Change < 0	25.7	5119	1	.55	3	.5
−200 ⩽ Change < −100	1.7	333	1	.3	1	.2
−400 ⩽ Change < −200	1.2	241	1	.1	3	1.0
−600 ⩽ Change < −400	.5	99	1	.1	1	.1
−900 ⩽ Change < −600	.3	69	2	.2	2	.2
−1600 ⩽ Change < −900**	.3	58	2	.3	2	.4
	100.0	19935				

* When the 77 additional observations having a percentage change in excess of 1600 are included in this stratum the submartingale (model 1, $\alpha = 1.0$) is the optimal model.

** When the 58 additional observations having a percentage change less than −1600 are included in this stratum the third order, $\alpha = .2$, provided the lowest mean absolute error and the second order, $\alpha = .2$, provided the lowest standard error of the estimate.

regression rule. Examination of the mean error terms also indicated mean reversion with this, and the next, stratification rule.

C. *The Normalized First Difference Stratification Rule*

The outcome of our tests utilizing the normalized first difference rule is presented in Table 3. The format of this table is identical to that of Table 2.

The outcome using this stratification rule supports the findings of the other two rules in that the best α-level for the strata on the extremes is less than one (not a submartingale process) and, for the sample as a whole, the best α-level would have been one. The mean absolute error measure produced results that were quite similar to those produced by the percentage change rule. Using mean absolute error, .8% of the sample on the income increase side had an α-level less than one and 27.7% on the decrease side had a best α-level less than one.

With the SEE measure, 52.9% of the sample had a best α-level less than one. The pattern of σ-levels is not as nearly consistent as with the MAE measure even though a larger proportion of the sample is not submartingale. Despite the increased instability of results that occured when using the standard error of the

TABLE 3

NORMALIZED FIRST DIFFERENCE STRATIFICATION RULE
PARAMETERS OF THE BEST PREDICTION MODEL FOR EACH STRATUM

			Mean Absolute Error		Standard Error of the Estimate	
1	2	3	4	5	6	7
Normalized First Difference	% of Sample	Sample Size	Order of Best Smoothing Model	Best Smoothing Constant for the Best Model	Order of Best Smoothing Model	Best Smoothing Constant for the Best Model
9 < Difference	.8	89	1	.9	1	.65
6 < Difference ⩽ 9	1.9	205	1	1.0	1	1.0
4 < Difference ⩽ 6	4.4	466	1	1.0	2	.1
2 < Difference ⩽ 4	16.7	1781	1	1.0	1	1.0
1 < Difference ⩽ 2	20.0	2136	1	1.0	3	.1
0 ⩽ Difference ⩽ 1	28.4	2977	1	1.0	1	1.0
−1 ⩽ Difference < 0	14.3	1531	1	.65	3	.45
−2 ⩽ Difference < −1	6.4	686	1	.45	1	.333
−4 ⩽ Difference < −2	4.4	478	1	.333	3	.333
−6 ⩽ Difference < −4	1.3	137	1	.1	1	.05
−9 ⩽ Difference < −6	.8	81	1	.3	1	.45
Difference < −9	.5	52	2	.2	2	.2
	100.0	10619				

estimate, the outcome of these tests further supports the results produced by the other stratification rules and the mean absolute error measure.

VI. SUMMARY AND CONCLUSION

The research reported in this paper represents the initial stage of research using a different approach to the examination of time-series properties of income. Previous studies have tested the entire sample without attempting an examination of subsets of series having homogeneous characteristics.

Three stratification rules were used to see if some consistent variations from the submartingale process occured in different strata. The best smoothing constant for first, second, and third order exponential smoothing models was then determined for each stratum to test for the existence of the submartingale.

Despite the limited scope of the research results being reported, three interesting properties of income time-series are suggested:

1. For the entire set of tests, a smoothing constant of one provided the best predictions. This supports other research that indicates that income time-series normally follow a submartingale or similar process.

2. However, series that fall in the outer strata have a best smoothing constant of less than one for predicting income in the period subsequent to the stratification period for all of the stratification rules and error measures used in the study. Thus, a substantial and identifiable portion of income time-series do not appear to follow a submartingale process.

3. The tendency of the best smoothing constant to change from one to some number less than one in those series included in the outer strata, together with an analysis of mean errors, indicates that income tends to revert to previous levels in the period subsequent to a substantial deviation from an operationally defined norm.

There are two important implications of the outcome of our study that are related to the Ball and Watts study discussed in Part II of this paper. First, the apparent temporal dependence of the series in the outer strata and the tendency of income to revert to previous levels make the conclusion that "all firms will, on the average, fail" [2, p. 665] questionable at best. Also, given Ball and Watts' definition of income smoothing, the evidence seems to provide a contradiction to their inference that management cannot be successful in efforts to smooth income.

We do not maintain that management does smooth income. The greater smoothness of series containing a material deviation from previous income levels may be management induced or inherent in the income determination process.

Perhaps the most interesting aspect of the results presented in this paper relates to the questions that are suggested but not investigated. A much broader and detailed examination of income time-series is called for in order to investigate further the phenomena suggested by this preliminary study. For example, the study can be expanded to include other stratification rules, prediction models, and both undeflated and deflated (rate of return measures) series. Other extensions include detailed examination of a random sample of series from each stratum to provide possible further evidence on the income smoothing controversy. Additionally, the inclusion of several periods subsequent to the stratification period in the tests might provide evidence on time-series behavior subsequent to a substantial disturbance.

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Appendix A

Exponential Smoothing Models

(1) *First-order smoothing*:

$$I_{t+1} = (I_t)\alpha + (1-\alpha)({}_1F_t), \qquad \text{where:}$$

I_t = actual income for period t;

${}_nF_t$ = forecasted income for period t. The prescript n indicates the order of the smoothing model.

α = smoothing constant

(2) *Second-order smoothing*:

$${}_2F_{t+1} = a_t + b_t, \qquad \text{where}$$

$$a_t = 2S_t(x) - {}_2S_t(X);$$

$$b_t = \left[\frac{\alpha}{1-\alpha}\right]\left[{}_1S_t(x) - {}_2S_t(x)\right];$$

$${}_1S_t(x) = \alpha I_t + (1-\alpha)\,{}_1S_{t-1}(x); \qquad \text{and}$$

$${}_2S_t(x) = \alpha S_t(x) + (1-\alpha)\,{}_2S_{t-1}(x),$$

where, ${}_nS_t$ = the smoothing function introduced in the nth order model.

(3) *Third-order smoothing*:

$$_3F_{t+1} = a_t + b_t + \frac{1}{2}c_t^2, \qquad \text{where}$$

$$a_t = 3S_t(x) - 3_2S_t(x) + _3S_t(x);$$

$$b_t = \left[\frac{\alpha}{2(1-\alpha)}\right]\left[(6-5\alpha)S_t(x) - 2(5-4\alpha)_2S_t(x) + (4-3\alpha)_3S_t(x)\right];$$

$$c_t = \left[\frac{\alpha^2}{(1-\alpha)^2}\right]\left[S_t(x) - 2_2S_t(x) + _3S_t(x)\right]; \qquad \text{and}$$

$$_3S_t(x) = \alpha_2S_t(x) + (1-\alpha)_3S_{t-t}(x).$$