

## LEARNING, CORPORATE CONTROL AND PERFORMANCE REQUIREMENTS IN VENTURE CAPITAL CONTRACTS

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We consider a two-period agency model in which both contracting parties have the skill to control production but one party's skill is unknown to both at contract signing. Interim information arrival reveals this skill to both parties and is used to determine who controls second period production. This model explains why venture capital contracts involve "bundling"—the combining of a risky claim for the venture capitalist with disproportionate control—and contain explicit covenants permitting passage of control to the venture capitalist following a poor performance by the entrepreneur. Additional features of venture capital contracts are also explained.

### I. INTRODUCTION

An important function of modern economies is the translation of new technologies and ideas into productive activities. A major channel for this has been the venture capital market.<sup>2</sup> There are currently about 550 venture capital firms and the annual size of venture capital investment is almost \$3 billion.<sup>3</sup> Many prominent firms, including Apple, Compaq, Digital Equipment Corporation, and Federal Express began as start-up companies funded by venture capitalists (VC's). VC's provide both capital and expertise that allow entrepreneurs to convert "raw" ideas into commercial ventures. Despite the importance of this market, the academic work on this topic is limited.

The salient features of a venture capital contract are as follows.<sup>4,5</sup>

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<sup>2</sup> Venture capitalists may be broadly categorized into two groups. One group consists of venture capitalists who specialize in providing "seed money" for start-up projects, and the other group consists of those who provide funds to relatively established firms, including those involved in leveraged buyouts. We focus on the former group.

<sup>3</sup> Source: CNN News.

<sup>4</sup> These stylized facts were gathered from a series of articles (Cooper and Carleton 1979; Dizard 1982; Marshall 1981; Silver 1981; and Tyebjee and Bruno 1984), as well as personal discussions with Mr. Golder of Golder Thoma Associates, one of the fifty largest U.S. venture capital firms.

<sup>5</sup> While each of these features can be found separately in other financial contracts, the venture capital contract is unique in that it is a "bundle" of all of these features. Thus, we recognize that a venture capital contract has features that are not necessarily *individually* idiosyncratic to that contract. For example, bond covenants frequently include provisions for passage of control contingent on performance that is

- (i) The entrepreneur cannot “walk away” after the first period and negotiate with another firm (no *de novo* financing).
- (ii) If the entrepreneur is relieved of productive control, then he is paid a fixed amount independent of his demonstrated skill and subsequent cash flows of the firm (“buyout” option for the VC).
- (iii) There is an explicit clause which stipulates that the entrepreneur should initially demonstrate a minimum skill level to be allowed to retain productive control in subsequent periods (“performance requirement”).
- (iv) If control remains with the entrepreneur, both the VC and the entrepreneur receive risky payoffs, which depend on the skill of the entrepreneur (“earnout” arrangement).

To confront these stylized facts about venture capital contracts, we develop an agency model in which there is pre-contract homogeneity in information structure and beliefs, but post-contract learning that affects the distribution of state-contingent payoffs as well as the question of who should control production. This model has the attractive feature that it captures important aspects of the start-up environment for entrepreneurs and also predicts the above-mentioned prominent features of venture capital contracts.

The model is as follows. There is a two-period economy consisting of competitive, risk neutral VC's and risk averse entrepreneurs. In the first period, the entrepreneur approaches a VC for capital to invest in a project that he “owns.” The project's payoff at the end of the second period depends both on the “skill” of the agent in “control” of the project and a choice of action by that agent. An agent's action choice is ex post unobservable to others. Both the entrepreneur and the VC can manage production.<sup>6</sup> The VC's skill is common knowledge but the entrepreneur's skill is unknown to all at the time of contract negotiation and capital investment. The project has two phases. The first period is a “developmental” phase during which a cash flow is generated and reinvested in the project. This cash flow reveals the entrepreneur's skill to the entrepreneur and to the VC. Based on this information, the VC determines whether to allow the entrepreneur to control production for the second period (the final and productive phase of the project) or to transfer control to himself. A cash flow is produced at the end of the second period and is shared by the entrepreneur and the VC in accordance with initially agreed-upon contractual provisions.

Our paper is related to the literature on venture capital as well as learning. The venture capital issue is analyzed in Cooper and Carleton (1979), Tyebjee and Bruno (1984) and Chan (1983), although none of these papers study the issues we do.<sup>7</sup> The

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unsatisfactory along some dimensions. Our aim is simply to explain why *all* of the features of a venture capital contract are found together.

<sup>6</sup> Contrast this with the standard agency formulation in which it is assumed that only the manager (entrepreneur) can manage the production process.

<sup>7</sup> Cooper and Carleton (1979) study how the project-linked payoff shares should be partitioned between the entrepreneur and the VC to ensure proximity of the investment policy to the value-maximizing policy. However, learning, moral hazard and optimal risk sharing are not examined. Tyebjee

role of learning in settings devoid of moral hazard has been explored in numerous papers. Harris and Holmstrom (1982) assume that risk neutral firms and risk averse workers both learn about the worker's productivity over time. Palfrey and Spatt (1985) study long-term insurance contracting in which both the insurer and the insured learn about the risk category of the insured. Models that combine moral hazard and learning include Lambert (1986), Holmstrom and Ricart i Costa (1986) and Lewis and Sappington (1987, 1989). In Lambert, the agent must be motivated to work to produce information about project profitability and then select the best project. In Holmstrom and Ricart i Costa, managerial skill is inferred over time and the agent's propensity to influence the perception of this skill sometimes causes him to overinvest; the principal copes by occasionally rationing capital. In Lewis and Sappington, the agent privately acquires information about a factor of production after the contract is negotiated. The problem studied is that of countervailing incentives, i.e., when the agent has an incentive to understate his private information for some of its realizations, and to overstate it for others.

What follows is in four sections. Section 2 has the notation, the assumptions and the model. Section 3 contains the analysis and results. Finally, Section 4 concludes. All proofs appear in an Appendix.

## 2. NOTATION, ASSUMPTIONS AND BASIC MODEL

### A. *Notation and Assumptions.*

(A1) *Time Horizon and Information Structure.* We assume that there are three dates of the model. At time zero, the entrepreneur and the VC sign the contract without knowledge of the entrepreneur's skill level,  $\gamma$ . Both parties share a common set of prior beliefs about  $\gamma$ , captured in the probability measure,  $F(\gamma)$ , defined over the interval  $[\gamma_1, \gamma_2] \subset \mathfrak{R}_+$ . Initial investment in the project takes place at time zero. At time one, the skill level of the entrepreneur,  $\gamma$ , is revealed.<sup>8</sup> Depending upon the realization of  $\gamma$ , control of the firm can be in the hands of either the entrepreneur or the VC. If the VC controls production, he does so with skill level normalized to  $\gamma = 1$ . A terminal cash flow is recovered at time two and shared by the entrepreneur and the VC according to a sharing rule agreed upon at time zero.

(A2) *Production Technology.* Production in the second period (revealed at time two) is completely described by the probability distribution of the terminal cash flow. Both the entrepreneur and the VC have available a set of actions  $A$ . For any  $a \in A$  and any  $\gamma \in [\gamma_1, \gamma_2]$ , the terminal cash flow is  $\gamma H$  with probability (w.p.)  $p(a) \in (0, 1)$  and  $\gamma L$  w.p.  $1 - p(a)$ , where  $\infty > H > L > 0$ , and  $p(a)$  is strictly increasing, twice continuously differentiable and concave.

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and Bruno (1984) is a purely descriptive analysis of venture capital allocation. Chan (1983) is concerned with how financial intermediaries help improve the allocation of investment capital, rather than details of the venture capital market.

<sup>8</sup> It is assumed the  $\gamma$  is revealed by the earnings of the firm,  $C(\gamma)$ , where  $C$  is a deterministic function with  $C' > 0$ .

(A3) *Preferences.* The entrepreneur's von Neuman-Morgenstern utility function is

$$U(\omega, a) = W(\omega) - V(a)$$

where  $W$  and  $V$  are defined on  $\mathfrak{R}_+$ ,  $V' > 0$ ,  $V'' \geq 0$ ,  $W' > 0$ ,  $W'' < 0$ . Further,  $W(\cdot)$  satisfies the Inada conditions  $W'(0) = \infty$  and  $W'(\infty) = 0$ . (For functions with one argument, primes denote partial derivatives and for functions with two or more arguments, subscripts.) If the VC controls the firm in the second period, then  $V(\cdot) = 0$ . As in Grossman and Hart (1983), we can now define the inverse function  $t(\cdot) \equiv W^{-1}(\cdot)$ , which is strictly increasing and strictly convex on  $\mathfrak{R}$ .<sup>9</sup> Doing this simplifies the later analysis by placing the util payments to the entrepreneur in dollar terms. We assume here that the entrepreneur is risk averse because, as an undiversified investor-manager, his wealth is largely committed to the project.

The VC is risk neutral. We make this assumption because VC's both have large portfolios of (diversifiable) projects and are owned by well-diversified investors. If the entrepreneur controls production in the second period, the VC only "cares" about his share of the terminal cash flow. If the VC is in control, then he cares not only about the terminal cash flow, but also about his effort disutility. We assume that the disutility for effort for the VC is  $t(V(a))$ , the pecuniary equivalent of disutility of the entrepreneur. We assume that the function  $V(a)$  is the same for both the entrepreneur and the VC, because we want to distinguish between the two agents *solely* along two dimensions—their disparate risk preferences and possibly different skill levels.<sup>10</sup>

(A4) *Endowments.* The entrepreneur has fully exhausted his endowment to invest in the project at the beginning of his planning horizon and still needs additional investment capital to "activate" the project. The VC has a sufficient initial capital endowment to provide the necessary funds.

(A5) *Form of the Contracts.* Since the project cash flow distribution has a two-point support for any skill  $\gamma$  and any action  $a$ , the second period contract stipulates a pair of (real) numbers representing the entrepreneur's payoffs for the two possible cash flow realizations. Let  $t(\chi)$  and  $t(\chi + \Delta)$  be the entrepreneur's payoffs corresponding to the terminal cash flow realizations  $\gamma L$  and  $\gamma H$  respectively.<sup>11</sup> Further, the contract must stipulate who controls production. Thus,

<sup>9</sup> Note that the existence of  $t(\cdot)$  is guaranteed by the fact that  $W(\cdot)$  is strictly increasing and continuous on  $\mathfrak{R}$ .

<sup>10</sup> The relationship we have assumed between the cost of effort to the entrepreneur and that to the VC is not the only one that will sustain our results. Clearly, the class of transformations that take us from one effort disutility function to the other such that our results are unchanged is quite large. We have chosen the transformation that is the most "neutral," i.e., one for which the monetary cost of a given effort level is the *same* for the entrepreneur and the VC. Thus, given our specification, the VC has no advantage relative to the entrepreneur in supplying effort.

<sup>11</sup> In actual venture capital contracts, initial levels of fixed and variable (payoff-contingent) compensation levels for the contracting parties are set when the contract is signed. At the conclusion of the initial stage of the project, these payoffs are adjusted on the basis of the revenues realized until then. In our context,  $\chi(\gamma)$  and  $\Delta(\gamma)$  should be interpreted as (the utilities of) the initial payoffs plus the adjustments after  $\gamma$  is revealed.

we can represent the *state-contingent contract* for the entrepreneur by  $\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}$ , where  $Z(\gamma) = 1$  if the entrepreneur is to control production in the second period and  $Z(\gamma) = 0$  if the VC is to control production in the second period. We assume that this contract is enforceable. An important feature of this contract is that ownership and control are specified independently. Thus, we are *not* requiring, *ex ante*, that the party with the majority equity ownership also control production. This is significant since an important feature of real world venture capital contracts is that the control that various contracting parties exercise is not always proportional to their equity representations.

(A6) *Market Structure.* The market for venture capital is perfectly competitive. Thus, VC's compete to give an entrepreneur his highest expected utility, subject to informational constraints and a reservation expected utility constraint for the VC that guarantees the VC a minimum expected utility of  $R$ . This reservation expected utility will be determined by the VC's investment, the VC's cost of funds and productive expertise.<sup>12</sup>

B. *The Model.*

(B1) *The Program.* The state contingent contract  $\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}$  solves

$$(1) \quad \text{Max}_{\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}} \int_{\gamma} EU(\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}) dF(\gamma)$$

subject to

$$(2) \quad \int_{\gamma} N(\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}) dF(\gamma) \geq R$$

where

$$(3) \quad EU(\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}) = p(\hat{a})[\chi(\gamma) + \Delta(\gamma)] + [1 - p(\hat{a})]\chi(\gamma) - V(\hat{a})Z(\gamma)$$

is the expected utility of the entrepreneur for skill  $\gamma$  and

$$(4) \quad N(\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}) = \gamma L + p(a^*)\gamma[H - L] - \{t(\chi) + p(a^*)[t(x + \Delta) - t(\chi)]\} \quad \text{for } Z(\gamma) = 1$$

$$(5) \quad N(\{\chi(\gamma), \Delta(\gamma), Z(\gamma)\}) = [1 - p(a^o)][L - t(\chi)] + p(a^o)[H - t(\chi + \Delta)] - t(V(a^o)) \quad \text{for } Z(\gamma) = 0$$

is the utility of the VC in state  $\gamma$ . In the above expressions,  $\hat{a}$  represents the optimal action, given the contract. This optimal action will depend on who controls production. When the entrepreneur is in control, we let  $\hat{a} = a^*$  and when the VC

<sup>12</sup> We assume that any cash generated in the first period,  $C(\gamma)$ , reduces funds needed from the VC, but does not eliminate the need for funds. This allows us to collapse the impact of  $C(\gamma)$  into the reservation utility of the VC, which is permissible because the VC is risk neutral.

is in control, we let  $\hat{a} = a^o$ . Henceforth, unless otherwise specified,  $*$  and  $^o$  will denote optimal values for the cases in which the entrepreneur and the VC are in control, respectively. We ignore discounting.

(B2) *Solution Strategy.* To solve this problem, we observe that any solution to our program must have the feature that it is *renegotiation-proof* (see Fudenberg and Tirole 1988) in the sense that once  $\gamma$  is revealed, there should be no incentive for mutually beneficial recontracting between the entrepreneur and the VC. This can only be the case if the state-contingent contract is *ex post* Pareto optimal. We impose this necessary condition by first solving for the optimal second period contract  $\{\hat{\chi}(\gamma, \bar{u}), \hat{\Delta}(\gamma, \bar{u}), \hat{Z}(\gamma, \bar{u})\}$ , subject to an arbitrary reservation utility constraint for the entrepreneur,  $\bar{u}$ , in state  $\gamma$ . Thus, the second period problem is that studied by Holmstrom (1979) and Grossman and Hart (1983). In the first period, then, we take the second period contract as a function of  $\{\gamma, \bar{u}\}$  and maximize the entrepreneur's expectation—taken with respect to  $F(\cdot)$ —over the function  $\{\bar{u}(\gamma), \gamma \in [\gamma_1, \gamma_2]\}$ , subject to the constraint that the VC is given at least his reservation utility.<sup>13</sup>

(B3) *Second Period Problem.* We now consider separately the case in which the entrepreneur is in control and that in which the VC is in control. We then compare the solutions to choose the optimal choice of control.

(i) *Entrepreneur in Control.* Conditional on the entrepreneur being in control, define  $N^*(\gamma, \bar{u})$  as the maximized expected payoff to the VC given the constraint that the entrepreneur attains a net expected utility of  $EU \geq \bar{u}$ , where  $EU$  is given by (3) with  $Z = 1$ . Because the Grossman and Hart (1983) *CDFC* condition holds here, we can use the first-order condition to characterize the entrepreneur's optimal choice of action,  $a^*$ . Thus,

$$(6) \quad N^*(\gamma, \bar{u}) = \text{Max}_{\chi, \Delta, a} N(\gamma, \chi, \Delta, a)$$

subject to

$$(7) \quad [1 - p(a)]\chi + p(a)[\chi + \Delta] - V(a) \geq \bar{u}$$

$$(8) \quad \Delta p'(a) = V'(a).$$

Note that Proposition 10 in Grossman and Hart (1983) implies (7) is an equality.

(ii) *VC in Control.* In this case the entrepreneur is “needed” only because he initially “owns” the project. Because the VC's skill parameter is unity, the maximum expected utility for the VC is (when the VC is in control, the solution is independent of the entrepreneur's ability,  $\gamma$ ):

$$(9) \quad N^o(\bar{u}) = \text{Max}_{\chi, \Delta, a} N(\bar{u}) = [1 - p(a)][L - t(\chi)] + p(a)[H - t(\chi + \Delta)] - t(V(a))$$

<sup>13</sup> We are essentially deriving the “contract curve” for the VC and the entrepreneur, given some  $\gamma$ . Mathematically, it is the same whether we hold  $\bar{u}$  fixed and solve for the maximum  $N$ , or hold  $N$  fixed and solve for the maximum  $\bar{u}$ . If we were to maximize  $\bar{u}$  given a fixed  $N$  and  $\gamma$  in the second period, we then have to optimize over  $N(\gamma)$  in the first period.

subject to

$$(10) \quad [1 - p(a)]\chi + p(a)[\chi + \Delta] \geq \bar{u}.$$

Here the VC's optimal action choice,  $a^o \in A$ , satisfies

$$(11) \quad p'(a^o)[\{H - L\} - \{t(\chi + \Delta) - t(\chi)\}] = t'(V(a^o)) \cdot V'(a^o).$$

(B4) *Choice of Control and the First Period Problem.* For a given  $\gamma$  and  $\bar{u}$ , the VC's second period expected payoff will be higher with the entrepreneur managing production, IFF

$$(12) \quad N^*(\gamma, \bar{u}) \geq N^o(\bar{u}).$$

If (12) is violated, the VC's second period expected payoff is higher if it manages production itself. That is,

$$(13) \quad \hat{Z}(\gamma, \bar{u}) = \begin{cases} 1 & \text{if } N^*(\gamma, \bar{u}) \geq N^o(\bar{u}) \\ 0 & \text{if } N^*(\gamma, \bar{u}) < N^o(\bar{u}). \end{cases}$$

To sum up, in the second period, given any  $\gamma$  and  $\bar{u}$ , ex post optimality completely specifies the contract and corresponding action choices,  $\{\hat{\chi}(\gamma, \bar{u}), \hat{\Delta}(\gamma, \bar{u}), \hat{Z}(\gamma, \bar{u}), \hat{a}(\gamma, \bar{u})\}$ . In solving the first period problem, we can take these functions as *given* and search for the optimal function  $\hat{u}(\gamma)$  to obtain the overall solution where each desired variable is an embedded function of  $\gamma$ . That is, in the first period the problem is to

$$(14) \quad \text{Max}_{\{\hat{u}(\gamma)\}} \int_{\Gamma_v} \hat{u}(\gamma) dF(\gamma) + \int_{\Gamma_e} \bar{u}(\gamma) dF(\gamma)$$

subject to

$$(15) \quad \int_{\Gamma_v} N^o(\bar{u}(\gamma)) dF(\gamma) + \int_{\Gamma_e} N^*(\gamma, \bar{u}(\gamma)) dF(\gamma) = R$$

where

$$(16) \quad \Gamma_e = \{\gamma \in [\gamma_1, \gamma_2] | N^*(\gamma, \bar{u}(\gamma)) \geq N^o(\bar{u}(\gamma))\}$$

$$\Gamma_v = \{\gamma \in [\gamma_1, \gamma_2] | N^*(\gamma, \bar{u}(\gamma)) < N^o(\bar{u}(\gamma))\}.$$

Thus, our solution precludes bilateral incentives to renegotiate the contract once  $\gamma$  is revealed.<sup>14</sup> However, the entrepreneur will desire to *unilaterally* seek outside funding for  $\gamma$ 's such that  $\hat{u}(\gamma) < \bar{u}(\gamma)$ , where  $\bar{u}(\gamma)$  is the utility that the entrepreneur

<sup>14</sup> In our model we do not allow the entrepreneur to have control over  $\gamma$ . If he did have control over  $\gamma$ , we would have moral hazard in the first period. This would significantly complicate the model. However, since most of our results depend only on the second period Pareto optimality of the contract, we conjecture that these results will be qualitatively sustained as long as the entrepreneur cannot misrepresent  $\gamma$ . This seems plausible given the close working relationship between the entrepreneur and the VC.

can receive with outside funding once  $\gamma$  is revealed at time one. We have assumed that the contract is binding, so that the entrepreneur cannot do this. In fact, as stated below, the entrepreneur is better off with the provision that he cannot seek *de novo* lending.

PROPOSITION 1. (*No de novo lending*) *The entrepreneur prefers to negotiate at time zero a provision in the contract that does not allow him to seek de novo lending at time one.*

Real world venture capital contracts usually stipulate that, after the project has been launched, the entrepreneur cannot “break away” from the VC and seek *de novo* financing prior to the conclusion of the contract. The desirability of such a provision is that there *will* be instances in which the entrepreneur will want to seek alternative financing *ex post*. However, if the VC were to permit this, the contract would need to be reformulated *ex ante* and this would lower the *entrepreneur’s* expected utility.

### 3. ANALYSIS AND FURTHER RESULTS

A. *Second Period Problem for Given*  $(\gamma, \bar{u})$ . We summarize below the results of the second period problem which facilitate the subsequent analysis of the overall problem. (Recall that \* and <sup>o</sup> designate control by the entrepreneur and the VC, respectively. When there is no ambiguity, we will drop the arguments  $(\gamma, \bar{u})$  from the functions  $(\chi, \Delta$  and  $a$ .)

LEMMA 1. *In the second period problem:*

(a) *If the VC is in control, an entrepreneur of ability  $\gamma$  is paid a fixed amount, regardless of the terminal cash flow:  $\chi^o(\bar{u}) = \bar{u}$  and  $\Delta^o(\bar{u}) = 0$ .*

(b) *If the entrepreneur is in control,*

(i)  $\Delta^* > 0$  and  $\gamma H - t(\chi^* + \Delta^*) > \gamma L - t(\chi^*)$ ,

(ii)  $\partial N^*(\gamma, \bar{u})/\partial \bar{u} = -\{t'(\chi^*) + p(a^*)[t'(\chi^* + \Delta^*) - t'(\chi^*)]\} < 0$ ,

(iii)  $\partial N^*(\gamma, \bar{u})/\partial \gamma > 0$ ,

(iv)  $\gamma > 1$ , and

(v) *given  $\bar{u}$ , if  $\gamma_a \neq \gamma_b$ , then either  $\chi^*(\gamma_a, \bar{u}) \neq \chi^*(\gamma_b, \bar{u})$  or  $\Delta^*(\gamma_a, \bar{u}) \neq \Delta^*(\gamma_b, \bar{u})$ .*

The reason that Lemma 1(a) holds is that the first best risk sharing arrangement involves the risk neutral VC completely insuring the risk averse entrepreneur against cash flow randomness. Because of moral hazard, this is not possible when the entrepreneur controls production. However, when the VC controls production, this optimal risk sharing arrangement also completely eliminates moral hazard, as a fixed payment to the entrepreneur makes him indifferent to the VC’s choice of action.

Clearly if the entrepreneur is in control,  $\gamma \geq 1$  (recall that 1 is the VC’s skill) to

ensure efficient allocation of productive control. The reason why  $\gamma > 1$  is moral hazard. When the VC controls production, there is not the moral hazard that exists when the entrepreneur controls production. Hence, for entrepreneurial skills exceeding the VC's skill by small values, it will still be efficient to let the VC assume control. For later use, denote

$$(17) \quad N^o(\bar{u}(\gamma)) = L + p(a^o)[H - L] - t(V(a^o)) - t(\bar{u}).$$

**B. The First Period Problem and Solution to the Overall Problem.** We now describe key properties of the solution to the program in (14) through (16). For notational ease, recall that  $\hat{u}(\gamma)$  is the optimal utility allocation for an entrepreneur of skill  $\gamma$ , regardless of who is in control.

**LEMMA 2.** *Ex ante (in the first period),  $\hat{u}(\gamma)$  is chosen in such a way that the marginal cost of  $\bar{u}$  to the VC is equated across all possible  $\gamma$  realizations.*

Our next result explains the “buyout” option in venture capital contracts.

**PROPOSITION 2.** *(Buyout Option) When the VC takes over productive control in the second period, the entrepreneur is paid a fixed amount regardless of his ability and the terminal cash flow.*

This is a stronger result than Lemma 1(a). It asserts that transfer of productive control to the VC means not only that the entrepreneur's compensation is nonrandom but also that this compensation does *not* depend on the entrepreneur's (known) skill level. The intuition is as follows. Because the entrepreneur is risk averse and the VC is risk neutral, optimal risk sharing requires that the VC bear all the cash flow risk. Since the VC is controlling production here, his payment to the entrepreneur of a fixed amount achieves both optimal risk sharing and efficient resolution of moral hazard. But why is this payment independent of  $\gamma$ ? The answer is that allocations across different  $\gamma$  realizations have to be Pareto efficient. From Lemma 2 we know that  $\hat{u}(\gamma)$  is chosen so that  $\partial \hat{N} / \partial \bar{u}$  is constant across all  $\gamma \in [\gamma_1, \gamma_2]$ . Since the entrepreneur takes no effort and receives a payment independent of the cash flow for all  $\gamma \in \Gamma_v$ , the only way to make  $\partial \hat{N} / \partial \bar{u} = \partial N^o / \partial \bar{u}$  constant for all  $\gamma \in \Gamma_v$  is to give the entrepreneur a constant  $\bar{u}$  for all such values of  $\gamma$ . And the only way to do this is to give him a fixed payment that does not depend on  $\gamma$ .<sup>15</sup> Next, we explain “performance requirements” in venture capital contracts. We shall assume henceforth that, associated with  $F$ , there is a density function  $f(\gamma)$  which is strictly positive over  $[\gamma_1, \gamma_2]$  and zero elsewhere.

**PROPOSITION 3.** *(Performance Requirement) The optimal venture capital contract contains the performance requirement that the entrepreneur retains control in the second period IFF his revealed skill level exceeds a critical value.*

<sup>15</sup> It is interesting to note that the independence of the entrepreneur's remuneration from  $\gamma$  when the VC is in control does *not* depend on the entrepreneur's risk aversion, i.e., it would be true even if the entrepreneur were risk neutral.

What is the incremental contribution of this proposition in light of Lemma 1(b)(iv)? Lemma 1 establishes that if the entrepreneur is optimally in control, then by necessity,  $\gamma > 1$ . However, it does not say whether the converse is true in the overall solution with  $\{\hat{u}(\gamma)\}$  simultaneously determined.<sup>16</sup> A more informative characterization is provided by the above proposition which says that every  $\gamma$  in  $\Gamma_e$  is strictly greater than any  $\gamma$  in  $\Gamma_v$ . That is, each of these two control sets is connected, the two sets are disjoint, and their union is  $[\gamma_1, \gamma_2]$ .

This striking theoretical characterization of the ex post allocation of productive control is interesting because it coincides with the key “performance requirement” covenant in real world venture capital contracts. Moreover, this proposition also rationalizes the bundling of a risky financial claim with a disproportionate control feature that is ubiquitous in venture capital contracts. The optimal control feature emerging from our analysis implies that the decision of who should control production is always socially efficient; whenever the VC takes over, the entrepreneur’s skill is below the critical level required. We now characterize the “earnout arrangement” as a feature of the optimal venture capital contract.

**PROPOSITION 4.** (*Earnout Arrangement*) *In skill realization states in which the entrepreneur is allowed to retain control in the second period, the bonus in utiles promised to him if the project yields a high terminal payoff is an increasing function of his revealed skilled level. That is,  $\Delta^*(\gamma_i) > \Delta^*(\gamma_j)$  for all  $\gamma_i, \gamma_j \in \Gamma_e$  and  $\gamma_i > \gamma_j$ .*

Since the entrepreneur’s effort is increasing in the bonus, more skilled entrepreneurs are induced to work harder. The intuition is that the marginal return to an increase in effort is higher for more skilled entrepreneurs.<sup>17</sup> Note, however, that this proposition is stated in terms of the entrepreneur’s *utility*. The fact that the entrepreneur enjoys a higher bonus utility in the second period if first period performance reveals him to be more skilled does *not* necessarily mean that he also gets a higher second period *pecuniary* bonus. That is, the difference between his utility given a high second period outcome and his utility given a low second period outcome can increase *without* the monetary reward for a high outcome going up. This proposition should, therefore, be interpreted as saying that the entrepreneur’s second period share of profits *depends* on  $\gamma$ , and not that it is necessarily increasing in  $\gamma$ . We now present an existence result.

**LEMMA 3.** *There exists a solution to the optimal control problem in (14) through (16), i.e., the first order optimality conditions are sufficient.*

Our analysis shows clearly that the optimal financial claim for the VC is to have a risky position in the firm; the VC gets  $\gamma H - t(\hat{\chi} + \hat{\Delta})$  in the high cash flow realization state and  $\gamma L - t(\hat{\chi})$  when there is a low cash flow realization, and neither of these payoffs is a constant regardless of who is in control.

<sup>16</sup> For example, in the absence of any monotonicity results, we cannot even tell whether either  $\Gamma_e$  or  $\Gamma_v$  is a connected set.

<sup>17</sup> Suppose that the payoffs in the high and the low states are  $\gamma + H$  and  $\gamma + L$  respectively. Then this proposition will not be true, although the others will still hold.

## 4. CONCLUDING REMARKS

Our preliminary theoretical look at the venture capital market has provided an explanation for numerous dominant characteristics of venture capital contracts. Although we have made simplifying assumptions, our results are robust with respect to extensions of the model in many directions. One natural extension is to introduce external uncertainties about production technology and market demand in addition to the uncertainty about an “internal” factor of input to production—entrepreneurial skill—that we have considered. This would allow for project abandonment after the first phase. It can be easily verified, however, that this does not change things much. All that is needed is that  $\gamma$  be correlated with the entrepreneur’s true skill, as long as information about true skill is symmetric (or else a revelation mechanism is required). That is, given the “signal”  $\gamma$ , there is an output  $y$ , with probability density function  $f(y, \gamma)$ , anticipated by both parties. Thus, there is no essential difference between the certainty case and the uncertainty case.

An issue not addressed in our model is the question of *why* the VC should act as a production controller that runs the firm itself in case the entrepreneur is insufficiently skilled. Could the firm simply not be sold off to another group of investors? To see why this may not be feasible or profitable, suppose the realization of the exogenous uncertainty mentioned above is private information for the VC and the entrepreneur. In the absence of a costless revelation mechanism, selling the project off to another group will be costlier (or infeasible if there is no revelation possible and Akerlofian (1970) market failure occurs) than the VC managing the project itself. Moreover, there is a natural benefit arising from the VC combining the roles of financier and production controller. If it were merely a pure financier (like a bank) that contracted with a consultant for the provision of productive expertise, there would be added moral hazard due to the consultant’s inclination to undersupply effort. This suggests that in settings where entrepreneurial skill is highly uncertain at the beginning and the role of a “backup” production controller is potentially significant, VC’s may have an advantage over banks in providing financing.<sup>18</sup> Future research along these lines may help to understand why VC’s specialize in “cradle to maturity” financing for entrepreneurs who are still learning how skilled they are.

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<sup>18</sup> This suggests that the most exciting prospect for future research in this area lies perhaps in an analysis of the conditions that lead to the simultaneous existence of banks and venture capitalists as credit sources. This may generate a more comprehensive theory of financial intermediary existence than those currently available. Moreover, it would explain why certain firms go to banks, some others go to VC’s and yet others go directly to the capital market. That is, we may see the beginning of an overall theory of financing.

## APPENDIX

PROOF OF PROPOSITION 1. If the entrepreneur could seek *de novo* financing at time one, then the maximization program in (1) through (5) would have to be solved subject to the additional constraint that  $\hat{u}(\gamma) \geq \bar{u}(\gamma) \forall \gamma$ . If this constraint is ever binding, the entrepreneur's expected utility will be lower with it. It is easy to show that there are cases in which the constraint is binding. For example, take  $W(\omega) = \sqrt{\omega}$ , so that  $t(y) = y^2$ . Further, assume  $p(a) = 1 - e^{-2a}$  and  $V(a) = a$ . In this case, we can show that  $\partial \hat{u}(\gamma) / \partial \gamma < 0$ . Thus, an entrepreneur who is more skilled gets a lower second period expected utility. A sufficiently highly skilled entrepreneur should, therefore, be able to approach a *de novo* lender at time one for better terms on the second period financing. Thus, the additional constraint will be binding in this case. This means that including a "no *de novo* financing" clause in the contract never makes the entrepreneur worse off but could make him strictly better off in some cases. Q.E.D.

PROOF OF LEMMA 1. (a) For any fixed  $a = \bar{a}$ , the optimization problem in (9) through (10) is

$$(A.1) \quad \text{Max}_{\chi, \Delta} \beta = L + p(\bar{a})[H - L] - \{t(\chi) + p(\bar{a})[t(\chi + \Delta) - t(\chi)]\} - t(V(\bar{a})),$$

where (10) must be satisfied. Clearly,  $N$  is maximized when  $[1 - p(\bar{a})]t(\chi) + p(\bar{a})t(\chi + \Delta)$  is minimized, since  $\bar{a}$  is held fixed. Note now that  $(\chi, \Delta)$  must be such that (10) holds tightly at the optimum. This implies that

$$(A.2) \quad \chi + p(\bar{a})\Delta = \bar{u}.$$

Now, by Jensen's inequality

$$\begin{aligned} [1 - p(\bar{a})]t(\chi) + p(\bar{a})t(\chi + \Delta) &> t(\chi[1 - p(\bar{a})] + p(\bar{a})[\chi + \Delta]) \\ &= t(\chi + p(\bar{a})\Delta) \\ &= t(\bar{u}) \quad \text{using (A.2)}. \end{aligned}$$

Thus, any schedule that makes  $\chi$  and  $\chi + \Delta$  different in the two states costs the VC more than setting the payment equal to  $t(\bar{u})$  in both states. Since  $\bar{a}$  was arbitrarily chosen, the argument also applies for  $\bar{a} = a^o$ .

(b) Part (i) follows from Grossman and Hart's Proposition 5 and their discussion in Section 4. Using (6) through (8), the Lagrangean for the constrained optimization problem when the entrepreneur is in control is as follows:

$$(A.3) \quad \beta = \gamma L + p(a)\gamma[H - L] - \{t(\chi) + p(a)[t(\chi + \Delta) - t(\chi)]\} \\ - \mu[\chi + p(a)\Delta - V(a) - \bar{u}] - \delta[V' - \Delta p']$$

where  $\mu$  and  $\delta$  are Lagrange multipliers. The first order conditions give

$$(A.4) \quad \mu = -\{t'(\chi^*) + p(a^*)[t'(\chi^* + \Delta^*) - t'(\chi^*)]\} < 0$$

$$(A.5) \quad \delta = p(a^*)[p'(a^*)]^{-1}[\mu + t'(\chi^* + \Delta^*)] > 0$$

$$(A.6) \quad V'(a^*) = \Delta^*p'(a^*)$$

$$(A.7) \quad \delta[V''(a^*) - \Delta^*p''(a^*)] = \{\gamma[H - L] + t(\chi^*) - t(\chi^* + \Delta^*)\}p'(a^*)$$

$$(A.8) \quad \bar{u} = \chi^* + p(a^*)\Delta^* - V(a^*),$$

where (A.4) is obtained from  $\partial\beta/\partial\chi = 0$ , (A.5) from  $\partial\beta/\partial\Delta = 0$ , (A.6) from  $\partial\beta/\partial\delta = 0$ , (A.7) from  $\partial\beta/\partial a = 0$ , and (A.8) from  $\partial\beta/\partial\mu = 0$ .

Now adding  $t'(\chi^* + \Delta^*)$  to both sides of (A.4), rearranging and combining with (A.5) yields

$$(A.9) \quad \delta = p(a^*)[p'(a^*)]^{-1}[1 - p(a^*)][t'(\chi^* + \Delta^*) - t'(\chi^*)].$$

Substituting for  $\delta$  from (A.9) into (A.7) gives us

$$(A.10) \quad p(a^*)[1 - p(a^*)][t'(\chi^* + \Delta^*) - t'(\chi^*)]\{V''(a^*) - \Delta^*p''(a^*)\} \\ = \{\gamma[H - L] + t(\chi^*) - t(\chi^* + \Delta^*)\}[p'(a^*)]^2,$$

where  $a^*$ ,  $\chi^*$  and  $\Delta^*$  are obtained by simultaneously solving (A.6), (A.8) and (A.10). The optimal solution then is  $\{\chi^*, \Delta^*, a^*\}$ , and  $N^*(\gamma, \bar{u}) = N(\chi^*, \Delta^*, a^*; \gamma, \bar{u})$ .

Results (ii) and (iii) are standard from the Lagrangean problem and we only sketch their proofs here:

Part (ii) follows from (A.4) as  $\partial N^*(\gamma, \bar{u})/\partial \bar{u} = \mu$ .

Part (iii) can be shown directly from the Envelope Theorem.

Part (iv) suppose that  $\gamma = 1$  for the entrepreneur. Then

$$(A.11) \quad N^o(\bar{u}) - N^*(1, \bar{u}) = p(a^o)[H - L] - t(V(a^o)) - t(\bar{u}) - p(a^*)[H - L] \\ + \{p(a^*)t(\chi^* + \Delta^*) + [1 - p(a^*)]t(\chi^*)\}.$$

Using (A.8) and a little algebra, we have

$$(A.12) \quad p(a^*)[\chi^* + \Delta^*] + [1 - p(a^*)]\chi^* = \bar{u} + V(a^*).$$

Since  $t(\cdot)$  is convex, Jensen's inequality yields

$$(A.13) \quad p(a^*)t(\chi^* + \Delta^*) + [1 - p(a^*)]t(\chi^*) > t(\bar{u} + V(a^*)).$$

Also, by the convexity of  $t(\cdot)$ , we know that

$$(A.14) \quad t(\bar{u} + V(a^*)) > t(\bar{u}) + t(V(a^*)).$$

Combining (A.13) and (A.14) produces

$$(A.15) \quad p(a^*)t(\chi^* + \Delta^*) + [1 - p(a^*)]t(\chi^*) > t(\bar{u}) + t(V(a^*)).$$

We can now express (A.11) as

$$\begin{aligned}
 N^o(\bar{u}) - N^*(1, \bar{u}) &= p(a^o)[H - L] - t(V(a^o)) - t(\bar{u}) \\
 &\quad - p(a^*)[H - L] + \{p(a^*)t(\chi^* + \Delta^*) + [1 - p(a^*)]t(\chi^*)\} \\
 &> p(a^o)[H - L] - t(V(a^o)) - p(a^*)[H - L] + t(V(a^*)) \\
 \text{(using (A.15))} \\
 &= \{p(a^o)[H - L] - t(V(a^o))\} - \{p(a^*)[H - L] - t(V(a^*))\} \\
 &\geq 0,
 \end{aligned}$$

where the last step follows from the fact that  $a^o$  is the maximizer of  $p(a)[H - L] - t(V(a))$ . Thus, since  $N^o(\bar{u}) - N^*(1, \bar{u}) > 0$  and  $dN^*/d\gamma|_{\bar{u}} > 0$ , we must have  $\gamma > 1$  (see (17)).

Part (v) given  $\bar{u}$ , suppose  $\gamma_a \neq \gamma_b$ , but  $\chi^*(\gamma_a, \bar{u}) = \chi^*(\gamma_b, \bar{u})$  and  $\Delta^*(\gamma_a, \bar{u}) = \Delta^*(\gamma_b, \bar{u})$ . From (A.6),  $a^*$  is determined by  $\Delta^*$ . Thus, the supposition implies the solution  $\{\chi^*, \Delta^*, a^*\}$  is the same for  $\gamma_a$  and  $\gamma_b$ . However, this contradicts (A.10) because (A.10) cannot hold as an equality for different  $\gamma$ 's with the same  $\{\chi^*, \Delta^*, a^*\}$ . Q.E.D.

PROOF OF LEMMA 2. Defining  $\alpha$  as the multiplier associated with (15), the objective function corresponding to the maximization problem in (14) through (16) is

$$\begin{aligned}
 \text{(A.16)} \quad &\int_{\Gamma_v} \bar{u}(\gamma) dF(\gamma) + \int_{\Gamma_e} \bar{u}(\gamma) dF(\gamma) \\
 &\quad - \alpha \left[ \int_{\Gamma_v} N^o(\bar{u}(\gamma)) dF(\gamma) + \int_{\Gamma_e} N^*(\gamma, \bar{u}(\gamma)) dF(\gamma) - R \right].
 \end{aligned}$$

Optimization yields

$$\text{(A.17)} \quad \alpha N^o_{\bar{u}}(\bar{u}(\gamma)) = 1 \quad \text{for } \gamma \in \Gamma_v.$$

$$\text{(A.18)} \quad \alpha N^*_{\bar{u}}(\gamma, \bar{u}(\gamma)) = 1 \quad \text{for } \gamma \in \Gamma_e.$$

Thus, the marginal cost of  $\bar{u}$  is constant for all  $\gamma$  at the optimum. Q.E.D.

PROOF OF PROPOSITION 2. Using (17) and noting that  $a^o$  is independent of  $\bar{u}$  yields

$$\text{(A.19)} \quad N^o_{\bar{u}}(\bar{u}(\gamma)) = -t'(\bar{u}(\gamma)).$$

Combining (A.17) and (A.19) yields

$$\text{(A.20)} \quad \alpha^{-1} = -t'(\bar{u}(\gamma)).$$

Because  $t(\cdot)$  is strictly increasing and convex, (A.20) implies that

$$\hat{u}(\gamma) = \bar{u} \text{ for all } \gamma,$$

where  $\bar{u}$  is a real-valued, scalar constant.

Q.E.D.

PROOF OF PROPOSITION 3. We shall prove the proposition by contradiction. For simplicity, we pick two arbitrary states  $\gamma_b$  and  $\gamma_a$  with the corresponding densities  $f(\gamma_b) \equiv f_b$  and  $f(\gamma_a) \equiv f_a$  and treat them as probabilities for  $\gamma_b$  and  $\gamma_a$ , respectively. The proof can be generalized easily to strips of  $\gamma$  around  $\gamma_b$  and  $\gamma_a$  with positive measure.

Suppose the presumed optimum with  $\gamma_b > \gamma_a$  is characterized as follows:

State	Control By	$\hat{u}(\gamma)$ for Entrepreneur	$N$ for VC
$\gamma_b$	VC	$\bar{u}_b$	$N^o(\bar{u}_b)$
$\gamma_a$	Entrepreneur	$\bar{u}_a$	$N^*(\gamma_a, \bar{u}_a)$

From Proposition 2,  $N^o(\cdot)$  is independent of  $\gamma$ . Let  $f \equiv \min \{f_b, f_a\}$ . Without loss of generality, suppose  $f = f_a \leq f_b$ . Consider the following new allocation of control,  $\hat{u}(\gamma)$  and  $N$ :

State	Control By	$\hat{u}(\gamma)$ for Entrepreneur	$N$ for VC
$\gamma_b$ w.p. $1 - f_a/f_b$	same allocation as in presumed optimum		
w.p. $f_a/f_b$	Entrepreneur	$\bar{u}_a$	$N^*(\gamma_b, \bar{u}_a)$
$\gamma_a$ w.p. 1	VC	$\bar{u}_b$	$N^o(\bar{u}_b)$

By construction the entrepreneur's expected utility does not change with the new allocation. The change in expected utility for the VC,  $\Delta EN$ , from the presumed optimum to the new allocation, is given by:

$$\begin{aligned} \Delta EN &= f_a \cdot \{N^*(\gamma_b, \bar{u}_a) - N^o(\bar{u}_b) + N^o(\bar{u}_b) - N^*(\gamma_a, \bar{u}_a)\} \\ &= f_a \cdot \{N^*(\gamma_b, \bar{u}_a) - N^*(\gamma_a, \bar{u}_a)\} \\ &> 0 \text{ [by } \gamma_b > \gamma_a \text{ and Lemma 1(b)(iii)]} \end{aligned}$$

which contradicts the presumed optimum.

Q.E.D.

PROOF OF PROPOSITION 4. The proof is divided in two parts. We first show that if  $\gamma_b \neq \gamma_a$ , then  $\Delta^*(\gamma_b, \bar{u}_b) \neq \Delta^*(\gamma_a, \bar{u}_a)$ , where  $\bar{u}_b \equiv \hat{u}(\gamma_b)$  and  $\bar{u}_a \equiv \hat{u}(\gamma_a)$ . Then we prove the proposition by contradiction.

Part (i). Suppose  $\gamma_b \neq \gamma_a$ , but  $\Delta^*(\gamma_b, \bar{u}_b) = \Delta^*(\gamma_a, \bar{u}_a)$ . From Lemmas 1 and 2,  $\partial N^*(\gamma, \hat{u}(\gamma))/\partial \bar{u} = -\{t'(\chi^*) + p(a^*)[t'(\chi^* + \Delta^*) - t'(\chi^*)]\}$  is invariant to  $\gamma$ . Since  $a^*$  is determined by  $\Delta^*$ , Lemma 2 holds for  $\gamma_b$  and  $\gamma_a$  only if  $\chi^*(\gamma_b, \bar{u}_b) = \chi^*(\gamma_a, \bar{u}_a)$ , which implies that  $\{\chi^*, \Delta^*, a^*\}$  is the same for  $\gamma_b$  and  $\gamma_a$ . Therefore,  $\bar{u}_b = \bar{u}_a$ . However, if  $\bar{u}_b = \bar{u}_a$  and  $\gamma_b \neq \gamma_a$ , Lemma 1(b)(v) implies that  $\{\chi^*, \Delta^*, a^*\}$  cannot be the same for  $\gamma_b$  and  $\gamma_a$ , a contradiction.

Part (ii). Now suppose  $\gamma_b > \gamma_a$ , but  $\Delta^*(\gamma_b, \bar{u}_b) < \Delta^*(\gamma_a, \bar{u}_a)$ . For ease of notation, define  $\Delta_j^* \equiv \Delta^*(\gamma_j, \bar{u}_j)$  and  $\chi_j^* \equiv \chi^*(\gamma_j, \bar{u}_j)$  for  $j = a, b$ . Also, recall the

definitions of  $f_b$  and  $f_a$  used in the proof of Proposition 3. The allocation of the presumed optimum is as follows:

State	Payoffs	$\hat{u}(\gamma)$ for Entrepreneur	$N$ for VC
$\gamma_b$	$\chi_b^*, \Delta_b^*$	$\bar{u}_b$	$N^*(\gamma_b, \bar{u}_b)$
$\gamma_a$	$\chi_a^*, \Delta_a^*$	$\bar{u}_a$	$N^*(\gamma_a, \bar{u}_a)$

Without loss of generality, suppose  $f_b \geq f_a = f \equiv \min \{f_b, f_a\}$ . Consider the following proposed new allocation:

State	Payoffs	$\hat{u}(\gamma)$ for Entrepreneur	$N$ for VC
$\gamma_b$ w.p. $1 - f_a/f_b$	same allocation as in presumed optimum		
w.p. $f_a/f_b$	$\chi_a^*, \Delta_a^*$	$\bar{u}_a$	$N^*(\gamma_b, \bar{u}_a)$
$\gamma_a$ w.p. 1	$\chi_b^*, \Delta_b^*$	$\bar{u}_b$	$N^*(\gamma_a, \bar{u}_b)$

By construction, the entrepreneur's expected utility would not change with the new allocation. The change in expected utility for the VC,  $\Delta EN$ , from the presumed optimum to the new allocation, is given by:

$$\Delta EN = f_a \cdot \{N^*(\gamma_b, \bar{u}_a) - N^*(\gamma_b, \bar{u}_b) + N^*(\gamma_a, \bar{u}_b) - N^*(\gamma_a, \bar{u}_a)\}.$$

But note that, for  $j, k = a, b$ ,

$$N^*(\gamma_j, \bar{u}_k) = \gamma_j p_k^* [H - L] + \gamma_j L - p_k^* t(\chi_k^* + \Delta_k^*) - [1 - p_k^*] t(\chi_k^*), \text{ where } p_k^* \equiv p(a^*(\Delta_k^*)).$$

Substituting into  $\Delta EN$  and simplifying, we have

$$\Delta EN = [\gamma_b - \gamma_a][p_a^* - p_b^*][H - L] > 0,$$

since by assumption  $\gamma_b > \gamma_a, H > L$ , and  $\Delta_a^* > \Delta_b^*$  implies  $p_a^* > p_b^*$ , a contradiction to the presumed optimum. Q.E.D.

PROOF OF LEMMA 3. The maximized value of the Hamiltonian corresponding to the augmented objective function (A.16) in this optimal control problem is linear and hence quasi-concave in  $\bar{u}(\gamma)$  for any given  $\gamma$ . The claim now follows from the Seierstad and Sydsaeter sufficiency theorem for optimal control problems with constraints on state and control variables (see Kamien and Schwartz 1981).

Q.E.D.

REFERENCES

AKERLOF, GEORGE A., "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics* 84 (August 1970), 488-500.  
 CHAN, YUK-SHEE, "On the Positive Role of Financial Intermediation in Allocation of Venture Capital in a Market with Imperfect Information," *Journal of Finance* 38 (December 1983), 1543-1568.  
 COOPER, IAN AND WILLARD CARLETON, "Dynamics of Borrower-Lender Interaction: Partitioning the Final Payoff in Venture Capital Finance," *Journal of Finance* (May 1979), 517-529.

- DIZARD, JOHN W., "Do We Have Too Many Venture Capitalists?" *Fortune* (October 4, 1982), 106-112.
- FUDENBERG, DREW AND JEAN TIROLE, "Moral Hazard and Renegotiation in Agency Contracts," M.I.T. working paper, February 1988.
- GROSSMAN, SANFORD AND OLIVER D. HART, "An Analysis of the Principal-Agent Problem," *Econometrica* 51 (January 1983), 7-36.
- HARRIS, MILTON AND BENGT HOLMSTROM, "A Theory of Wage Dynamics," *Review of Economic Studies* 49 (July 1982), 315-333.
- HOLMSTROM, BENGT, "Moral Hazard and Observability," *Rand Journal of Economics* 10 (1979), 74-91.
- AND JOAN RICART I COSTA, "Managerial Incentives and Capital Management," *Quarterly Journal of Economics* CI-4 (November 1986), 835-860.
- KAMIEN, MORTON AND NANCY SCHWARTZ, *Dynamic Optimization* (New York: North Holland Publishing Company, 1981).
- LAMBERT, RICHARD A., "Executive Effort and Selection of Risky Projects," *Rand Journal of Economics* 17 (spring 1986), 77-88.
- LEWIS, TRACY R. AND DAVID E. M. SAPPINGTON, "Inflexible Rules in Incentive Problems," *American Economic Review* 79 (March 1989), 69-84.
- AND ———, "Countervailing Incentives in Agency Problems," *Econometrica* (forthcoming).
- MARSHALL, MARTIN, "Time is Ripe for Entrepreneurs," *Electronics* 54 (August 1981), pp. 93-104.
- PALFREY, THOMAS R., AND CHESTER S. SPATT, "Repeated Insurance Contracts and Learning," *The Rand Journal of Economics* 16 (autumn 1985), 356-367.
- SILVER, DAVID A., "Blowing the Whistle," *Venture* 3 (November 1981), 32-33.
- TYEBJEE, TYZON AND ALBERT BRUNO, "A Model of Venture Capitalist Investment Activity," *Management Science* 30 (September 1984), 1051-1066.